Symmetry of the Relativistic Quantum Mechanical Harmonic Oscillator and the Antinucleon Spectrum in Nuclei

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As is well-known, the nonrelativistic spherical harmonic oscillator has degeneracies in addition to those due to rotational invariance. The energy spectrum depends only on the *total* harmonic oscillator quantum number N, $N = 2n + \ell$, where n is the radial quantum number and ℓ is the orbital angular momentum. Hence, the states with $\ell = N, N - 2, \ldots$ 0 or 1 have the same energy. These degeneracies are produced by an U(3) symmetry which has been influential in connecting the shell model with collective motion. Also the energy does not depend on the orientation of the spin and hence the nonrelativistic harmonic oscillator has a spin symmetry as well.

Since relativistic models of nuclei are now so prevalent [1], we can ask if U(3) symmetry resides in the relativistic harmonic oscillator. Indeed the Dirac Hamiltonian for which the scalar and vector potentials are equal and harmonic has been solved analytically and is invariant under a spin symmetry [2, 3]. Just as for the nonrelativistic harmonic oscillator, the spherically symmetric relativistic harmonic oscillator energy spectrum depends only on the total harmonic oscillator quantum number N, although the energy spectrum for the relativistic harmonic oscillator spectrum in general does not have a linear dependence on N as does the nonrelativistic harmonic oscillator. In Fig. 1 we plot the spherical harmonic oscillator Dirac binding energies E_D, the solid curve, as a function of N. We chose the harmonic oscillator strength to fit the lowest eigenenergies of the spectrum of an antiproton outside of ^{16}O in the relativistic mean field approximation [4]. The dashed curve is E_D in the pertubation approximation for large antiproton mass, which reduces to the nonrelativistic approximation. The short-dashed curve is E_D in the asymptotic limit for small mass and is the highly relativistic limit. Clearly the eigenenergies are in the relativistic asymptotic regime and not the linear regime of the nonrelativistic harmonic oscillator.

The fact that the relativistic spectrum depends only on N does suggest that the relativistic harmonic oscillator has an U(3) symmetry. We have shown that there is indeed a U(3) symmetry and we have derived the generators which commute with the Dirac Hamiltonian [5].

If speculation that an antinucleon can be bound inside a nucleus is valid [4], the antinucleon spectrum will have an approximate spin symmetry, and most likely an approximate U(3) symmetry, because the vector and scalar potentials are approximately equal and are very strong [2].

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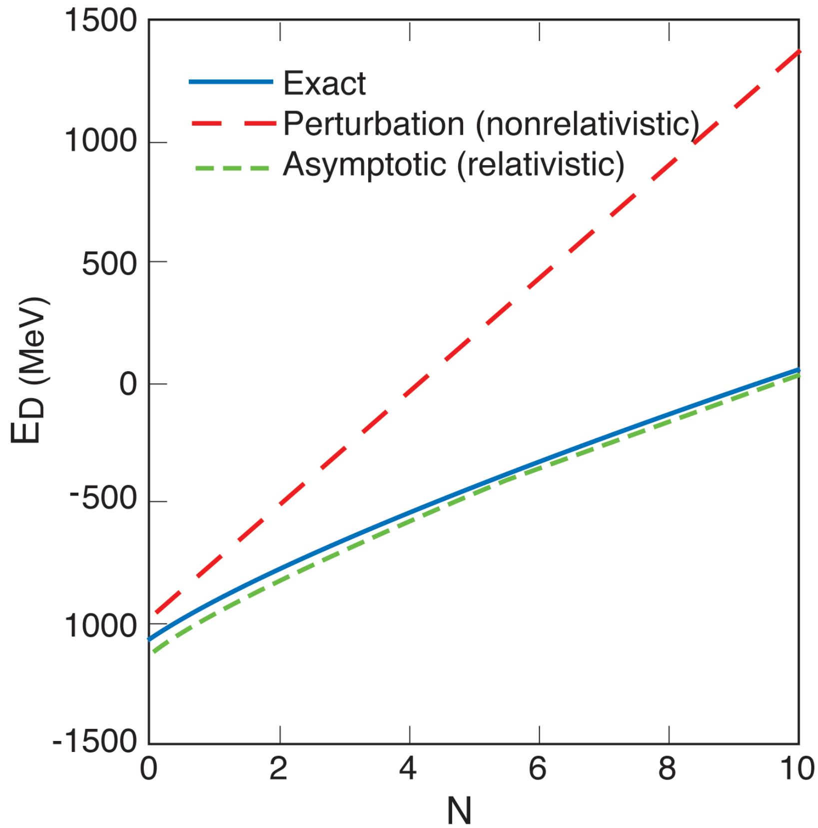

Fig. 1.
The Dirac binding energies, E_D, for the spherical harmonic oscillator as a function of N. The exact energies are the solid line, the perturbation approximation (nonrelativistic) is the dashed line, and the asymptotic approximation (relativistic) is the short dashed line.